Workshop ”Copulae in Mathematical and Quantitative Finance”

Preliminary Book of Abstracts

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1
Algebraic properties of copulas defined from matrices

Cécile Amblard*, Stéphane Girard, Ludovic Mennetau

We propose a new family of copulas, defined by:

$$S_\phi(u,v) = {}^t\phi(u)A\phi(v), \ \ (u,v) \in [0,1]^2,$$

where ϕ is a function from $[0,1]$ to $\mathbb{R}^p$ and A is a $p \times p$ matrix. Let us remark that if $p = 2$ and A is a diagonal matrix, then S_ϕ reduces to the family proposed in [1]. As a consequence, S_ϕ can be seen as an extension of this former family to arbitrary matrices.
First, we shall give sufficient conditions on A and ϕ to obtain copulas. Then, we shall establish the dependence and symmetry properties of this family of copulas. Finally, we shall study the stability properties of S_ϕ with respect to the operator $*$ (presented for instance in [2], p. 194) as well as other algebraic properties.

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Cécile Amblard
Université Grenoble 1
e-mail: Cecile.Amblard@imag.fr
Stéphane Girard
INRIA Grenoble Rhône-Alpes
e-mail: Stephane.Girard@inria.fr
Ludovic Mennetau
Université Montpellier 2
e-mail: mennet@math.univ-montp2.fr

2

An efficient importance sampling algorithm for copula models in insurance

Philipp Arbenz, Mathieu Cambou*, Marius Hofert

We introduce a simple importance sampling algorithm for copula models. The method improves Monte Carlo estimators in the case where the functional of interest depends mainly on the behaviour of the random vector when at least one of the components is large. Such problems often arise in financial and insurance dependence models, where distorted expectations of heavy tailed distributions are involved. The importance sampling framework we propose is general and can be implemented for all classes of copula models from which sampling is possible. Particular cases of copula models are then proposed for ease of sampling and optimizing the proposal distribution.

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Philipp Arbenz, RiskLab, D-MATH, ETH Zürich, e-mail:philipp.arbenz@math.ethz.ch
Mathieu Cambou, EDMA, EPFL, Lausanne, e-mail:mathieu.cambou@epfl.ch
Marius Hofert, RiskLab, D-MATH, ETH Zürich, e-mail:marius.hofert@math.ethz.ch

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A new copula approach for high-dimensional real world portfolios

Wolfgang Aussenegg and Christian Cech*

The aim of this paper is to generate a new copula based Value at Risk (VaR) approach that can be applied to high-dimensional real world portfolios. Current VaR copula models typically only can deal with portfolios consisting of just a few risk factors. They are, therefore, not suitable for practical applications. This paper tries to fill this gap by presenting a new parsimonious and fast calibration algorithm for the Student *t* copula model. The new approach provides for the first time the possibility to generate VaR estimates based on Student *t* copulas for high-dimensional real world portfolios.

A portfolio of 21 different financial assets and three additional VaR models (Variance-Covariance, Gaussian copula, and historical simulation) are used to evaluate the suitability of this new Student *t* copula approach. Almost 20 years of data are used to conduct an out-of-sample hit test based on a rolling window of 250 trading days for model calibration. The results of the hit test reveal that the model performance is highly affected by volatility clustering. Thus, all models perform poorly based on empirical returns, a fact that can be attributed to the underestimation of risk during the financial crisis in 2008. The new Student *t* copula approach and the historical simulation model perform best, whereas the Variance-Covariance model performs worst in this environment.

Accounting for volatility-clustering and applying the models on GARCH(1,1)-innovations rather than on empirical returns considerably improves the performance, with the exception of the Variance-Covariance model. Overall, the weaknesses of the Variance-Covariance model stems from three sources: (a) An inappropriate modeling of (univariate) return distributions, (b) an inappropriate modeling of the ‘dependence structure’ (i.e. the copula), and (c) not accounting for volatility clustering. The proposed new Student *t* copula approach tends to overcome these weaknesses when volatility clustering is accounted for. It is, therefore, a quite promising parametric model alternative for the Variance-Covariance model.

Wolfgang Aussenegg, Vienna University of Technology
e-mail:waussen@pop.tuwien.ac.at
Christian Cech, University of Applied Sciences bfi Vienna
e-mail:christian.cech@fh-vie.ac.at

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Semi-Parametric Measure of Temporal Dependence through Copulas

Kazim Azam*, Gianna Boero and Michael Pitt

Copulas have mostly been used to model interdependence among random variables. We set out a semi-parametric based stationary markov chain by a copula which captures the scale-free temporal dependence and tail dependence of the processes. These markov processes can model the non-linear dependency found among financial and economic applications. Chen, Wu and Yi (2009) show such processes generated via Clayton, Gumbel and Student-t copulas to be all geometric ergodic. We make no assumption regarding the marginal distribution of the random variables but simply extract the information contained through order statistic which provides the sufficient statistic for the parameter of interest (tail dependency etc.). The purpose of this paper is to extend the methodology presented by Hoff (2008) for time-series purposes. Our methodology can be employed to any family of copulas and can deal with continuous, discrete data and account for missing data. We present a general estimation scheme using bayesian specifications where we simply use the conditional distribution of a copula and a MCMC algorithm is presented. The approach is invariant to any copula family. Some simulations are presented in the end along with a Value-at-risk example.

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Kazim Azam, Dept. of economics, University of Warwick, kazim.azam@warwick.ac.uk
Dr. Gianna Boero, Dept. of economics, University of Warwick, g.boero@warwick.ac.uk
Dr. Michael Pitt, Dept. of economics, University of Warwick, m.pitt@warwick.ac.uk

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Calibration of Copulas and VaR estimates

Mária Bohdalová*, Michal Greguš

Calibration techniques can be based on the connection between rank correlation and certain one-parameter bivariate copulas. This correspondence allows for easy calibration of the parameter. In this paper we use more general numerical calibration techniques that are based on maximum likelihood estimation (MLE). Using this approach we want to estimate VaR of the fictive portfolio using Monte Carlo simulations. We focus on modelling the interdependence between different types of risk factor return. We suppose that the risk factor returns have some assumed marginal distributions, which need not be identical, and their dependency is modelled with copulas.

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Mária Bohdalová
Comenius University in Bratislava, Faculty of Management
e-mail:maria.bohdalova@fm.uniba.sk
Michal Greguš
Comenius University in Bratislava, Faculty of Management
e-mail:michal.gregus@fm.uniba.sk

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6
Hierarchical Kendall copulas – A new class of multivariate dependence models

Eike Christian Brechmann

While there is substantial need for dependence models in high dimensions, most existing models strongly suffer from the curse of dimensionality and barely balance parsimony and flexibility. In this talk, the new class of hierarchical Kendall copulas is proposed which tackles these problems. Constructed with flexible copulas specified for groups of variables in different hierarchical levels, hierarchical Kendall copulas are able to model complex dependence patterns without severe restrictions.

In particular, let $U_1,\ldots,U_n$ be uniform random variables, C_0 a d-dimensional copula and $C_1,\ldots,C_d$ copulas of dimension $n_1,\ldots,n_d$, where $n=\sum_{i=1}^{d} n_i$. Moreover, for all $i=1,\ldots,d$ let K_i denote the Kendall distribution function corresponding to C_i, that is the multivariate probability integral transform $K_i(t) := P(C_i(U_{m_{i-1}+1},\ldots,U_{m_i}) \le t)$ for $t \in (0,1)$, where $m_i = \sum_{j=1}^{i} n_j$ for $i=0,\ldots,d$. Then the two level hierarchical Kendall copula model is defined as follows:

1. $(U_{m_{i-1}+1},\ldots,U_{m_i}) \sim C_i \; \forall i \in \{1,\ldots,d\}$,
2. $(V_1,\ldots,V_d) \sim C_0$, where $V_i := K_i(C_i(U_{m_{i-1}+1},\ldots,U_{m_i})) \; \forall i \in \{1,\ldots,d\}$.

The definition can also easily be extended to an arbitrary number of levels.

The cluster copulas $C_1,\ldots,C_d$ as well as the nesting copula C_0 can be chosen independently from any class of copulas so that no dependence restrictions such as for competing models are required. While the grouped Student-t copula is limited to elliptical dependence, hierarchical Archimedean copulas are restricted to the class of Archimedean copulas as building blocks and must always have stronger dependence within clusters than among clusters.

The talk explicitly discusses properties as well as inference techniques for hierarchical Kendall copulas, in particular, simulation, estimation and model selection. A closed-form sampling algorithm is derived for Archimedean copulas, while for general copulas an approximative method is proposed. For estimation, a sequential and a joint approach are discussed and compared in an extensive simulation study. A substantive application to German stock returns finally shows that hierarchical Kendall copulas perform very well, out-of- as well as in-sample.

Eike Christian Brechmann
Technische Universität München
e-mail: brechmann@ma.tum.de

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A copula-based analysis of false discovery rate control under dependence assumptions

Roy Cerqueti, Mauro Costantini and Claudio Lupi*

The False Discovery Rate (FDR) was proposed in Benjamini and Hochberg (1995) as a powerful approach to the multiplicity problem that does not require strong control of the familywise error rate (FWER). The original approach was developed for independent test statistics and was lately extended to generally dependent statistics in Yekutieli (2008). In this paper we extend the existing results by showing that specific copulæ can be used to represent the dependence structure among classes of univariate statistics that leads to FDR control. In particular, within a fairly general setting we use Liebscher (2008, 2011) and Müller and Scarsini (2005) and show that FDR control is ensured when the dependence existing among the statistics is described by asymmetric Archimedean copulæ. In deriving our results we extend Yekutieli (2008).

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Roy Cerqueti
University of Macerata, Dept. of Economic and Financial Institutions
e-mail: roy.cerqueti@unimc.it
Mauro Costantini
Brunel University, Dept. of Economics and Finance
e-mail: Mauro.Costantini@brunel.ac.uk
Claudio Lupi
University of Molise, Dept. of Economics, Management, and Social Sciences
e-mail: lupi@unimol.it

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Model selection for regular vine models with applications

Claudia Czado

Pair copula constructions (PCC) allow to build very flexible multivariate distributions called regular vines. This class is very useful for modeling multivariate data in economics and finance, since it can capture non symmetric and different tail dependencies for different pairs of variables separately. Vine models are characterized by a sequence of linked trees called a vine tree structure, bivariate copula families and marginal distributions. Two often studied subclasses are C- and D-vines. The multivariate Gauss and t-distribution are special cases. Previously the development of efficient estimation methods were the focus (see for example [1] or [2]). Since the class of regular vines is very large, model selection is vital. The results of [3] provide a fast selection method, where trees are sampled sequentially using aglorithms for graphs with weights. In addition copula families and the corresponding parameters are selected by AIC. Bayesian MCMC methods were developed in in [4] and [5]) to detect conditional independencies in a given D-vine. The more general vine tree structure selection model problem is tackled in [6] using reversible jump MCMC. Methods will be illustrated by applications to financial times series.

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Claudia Czado, Technische Universität München, e-mail:cczado@ma.tum.de

9
Modeling self-selection bias using copulas: a Bayesian approach

Luciana Dalla Valle

This work illustrates a flexible methodology to impute missing answers, which are very frequent in survey analysis. This approach is based on copula functions with Bayesian parameter estimation applied to the Heckman's self-selection method.

In the analysis of surveys researchers pay special attention to biases due to non sampling errors that might affect the reliability of results. Among these errors, this work focuses on item non responses. Suppose that we know the values of a target variable observed on a self-selected sample. In order to estimate a parameter of the population or a model showing the relationships among the variables of interest, we need to consider not only the observed values, but also the unobserved values of the set of non self-selected units. For this purpose, Heckman proposed a two-step model using the target variable values observed on a self-selected sample (hence not random), and the values of a set of auxiliary variables known for all units of population. However, the model assumption of bivariate normality is quite restrictive and is not suitable for all types of data. Therefore, this work proposes a generalisation of Heckman's model with the introduction of copulas. This methodology is attractive since it allows to assume different distributions for the marginals and to express various dependence structures according to the different types of copulas in a Bayesian framework. The methodology is illustrated through an application to real data, where the parameters are estimated using the Bayesian approach.

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Luciana Dalla Valle
School of Computing and Mathematics, Plymouth University
e-mail:luciana.dallavalle@plymouth.ac.uk

10

Characterization of all copulas associated with non-continuous random variables

Enrique de Amo*, Manuel Díaz Carrillo, Juan Fernández Sánchez

For any integer $n \geq 2$, an n-dimensional copula is the restriction to the unit n-cube $[0,1]^n$ of a multivariate cumulative distribution function whose margins are uniform on $[0,1]$. They were introduced by Sklar in 1959 (see [2]), as the answer to a question posed by M. Fréchet, and they allow to represent a joint distribution of random variables as a function of marginal distributions. Precisely, Sklar enunciated that if H is the joint distribution function of n random variables $X_1, \ldots, X_n$, and $F_1, \ldots, F_n$ are the distribution functions of $X_1, \ldots, X_n$, respectively, then there exists an n-dimensional copula C such that

$$H(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$$

for all $x_1, \ldots, x_n \in \mathbb{R}^n$. Such C is uniquely determined on $\operatorname{Ran}(F_1) \times \cdots \times \operatorname{Ran}(F_n)$.

Nowadays, this result is known as Sklar's theorem. The first proof of this theorem (in the bidimensional case) was published in 1974 by Schweizer and Sklar [3]. The wide variety of different proofs of Sklar's theorem are based on techniques that go from those which are purely probabilistic to others which are more analytic, as the rearrangement inequalities of Hardy-Littlewood and Riesz. In this communication we consider the bidimensional case and, following the way traced by [3], we present, as the main result, a representation of all copulas C satisfying Sklar's theorem for a given bidimensional distribution H with margins F and G.

The method we use, which we name the E-process, consists of the construction of doubly stochastic measures to obtain these copulas C. The procedure to obtain C is constructive and it is based on patchwork techniques.

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Enrique de Amo, Universidad de Almería, e-mail:edeamo@ual.es
Manuel Díaz Carrillo, Universidad de Granada, e-mail:madiaz@ugr.es
Juan Fernández Sánchez, Universidad de Almería, e-mail:juanfernandez@ual.es

11
Modeling joint price spikes of Australian electricity markets

Michael Eichler, Oliver Grothe, Hans Manner* and Dennis Tuerk

We propose a model for the joint distribution of so called prices spikes, rather arbitrarily defined as prices exceeding 100, of half hourly electricity spot prices of markets in Australia. Marginally, the most extreme spikes can be modeled using extreme value theory, whereas moderate price spikes are more appropriately modeled by a Weibull distribution. The dependence between the spikes is then modeled by using copulas and three important issues need to be addressed: i) We are only interested in modeling the dependence of a specific region of the joint distribution while ignoring all non-spike prices. ii) The value of the threshold defining a spike is rather arbitrary so the model should be robust to that choice. iii) Given the market structure in Australia the dependence between markets may not always be symmetric, meaning that $C(u,v) \neq C(v,u)$. This last point can be captured by relying on, among other potential models, the asymmetric Gumbel copula, which does not only allow for this feature while nesting the standard Gumbel copula, but that also falls into the class of extreme value copulas. The first two points are acknowledged by explicitly taking into account observations that are no joint spikes using censored maximum likelihood estimation. We show that this estimation method allows for very large efficiency gains compared to maximum likelihood estimation based only on joint spikes. Furthermore, the exact choice of the threshold characterizing spikes becomes less relevant since the information in the data for prices not exceeding the threshold is not ignored.

We provide density forecasts for joint price spikes and evaluate the out-of-sample fit of our model based on statistical and economic criteria.

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Michael Eichler, Maastricht University
Oliver Grothe, University of Cologne
Hans Manner, University of Cologne
Dennis Tuerk, Maastricht University

12
Copula-based graduation and simulation of mortality tables

Arturo Erdely

Graduation of mortality tables is intended to estimate probabilities of death by means of an age-dependent function based on sample information of observed death rates. In actuarial practice this problem has been solved traditionally using deterministic smoothing techniques without a statistical model to provide, in addition to a model for the trend, a quantification of the extent of variability associated with the data. But from a modern risk management point of view, it is necessary a statistical methodology that models simultaneously both the problem of graduation and that of over/under estimation. A copula-based alternative will be presented for such purpose, which in addition may be used for simulation of scenarios for mortality risk assessment, with an application to real mortality data from Mexican insurance companies.

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Arturo Erdely
Actuarial Science Program, Facultad de Estudios Superiores Acatlán – U.N.A.M.
e-mail: arturo.erdely@comunidad.unam.mx

13
An improved Kolmogorov-Smirnov test for copulas

Jean-David Fermanian*, Dragan Radulovic and Marten H. Wegkamp

The authors propose new goodness-of-fit test procedures for copulas, based on empirical copula processes and their nonparametric bootstrap counterparts. They extend the scope of the Kolmogorov-Smirnov test significantly by considering infinite sequences of boxes, or even all the partitions in the unit cube. Although the underlying empirical processes (indexed by convenient classes of functions) do not converge, it is proved we can approximate the p-values of these test statistics after bootstrapping, even for composite zero hypotheses. Some simulations and empirical applications on financial data show that the power of these new procedures are higher than those of the standard Kolmogorov-Smirnov test for copulas.

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Jean-David Fermanian
Crest-Ensae, Malakoff, France
Dragan Radulovic
Department of Mathematics, Florida Atlantic University
Marten H. Wegkamp
Department of Mathematics & Department of Statistical Science, Cornell University

14
Construction of Multivariate Copulas in n-Boxes

José M. González-Barrios* and María M. Hernández-Cedillo

In this talk we generalize to n dimensions the general frameworks for constructing new copulas by using patchwork assemblies inside a base copula. We first generalize the work of De Baets and De Meyer [3], and then we give an alternative proof of the rectangular patchwork proposed by Durante, Saminger-Platz and Sarkoci [5]. We also give a characterization of modular n dimensional functions which provide a methodology for constructing new n-increasing functions which may provide extensions of the rectangular patchwork to larger dimensions.

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José M. González-Barrios
Departamento de Probabilidad y Estadística, IIMAS - Universidad Nacional Autónoma de México, México
e-mail:gonzaba@sigma.iimas.unam.mx
María M. Hernández-Cedillo
Departamento de Probabilidad y Estadística, IIMAS - Universidad Nacional Autónoma de México, México
e-mail:magdaz@gmail.com

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Modeling Joint Extreme Events Using Multivariate Self-Exciting Jump Processes

Oliver Grothe, Volodymyr Korniichuk*, Hans Manner

We develop an approach to model the probability of multivariate joint extreme events conditional on the history of the past extreme events. To this end we extend a popular self-exciting peaks-over-threshold model to the multi-dimensional case. This is achieved by relying on Extreme Value copulas and Hawkes processes. In particular, we model joint extreme events as a point process with a conditional rate that incorporates the effect of past extremes events, both marginal and joint, using an extreme value copula. Furthermore, the dependence parameter of the copula is allowed to depend on a self-exciting process and is, therefore, time-varying. We discuss the properties of this model, treat its estimation and deal with testing the goodness-of-fit. We also develop an algorithm to simulate artificial data from the proposed model. As an application, we analyze joint extreme events of the USA and European financial markets and joint extreme events in European banking sector considering four major European banks.

Keywords: Extreme Value Copula, Extreme Value Theory, Hawkes processes, Peaks over Threshold.

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Oliver Grothe, University of Cologne, Department of Economic and Social Statistics.
Volodymyr Korniichuk, University of Cologne, Cologne Graduate School,
email: korniichuk@wiso.uni-koeln.de.
Hans Manner, University of Cologne, Department of Economic and Social Statistics.

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Vine Constructions of Lévy Copulas

Oliver Grothe, Stephan Nicklas*

Lévy copulas are the most natural concept to capture the jump dependence of multivariate Lévy processes. This approach translates the intuition and many features of the copula concept into a time series setting. A challenge which distributional and Lévy copulas face is to find flexible but still applicable models for higher dimensions. To overcome this problem, the concept of pair copula constructions has been successfully applied to distributional copulas. In this paper we develop the pair construction for Lévy copulas. Similar to pair constructions of distributional copulas, the pair construction of a d-dimensional Lévy copula consists of $d(d-1)/2$ bivariate dependence functions. We show that only $d-1$ of these bivariate functions are Lévy copulas, whereas the remaining functions are distributional copulas. Since there are no restrictions concerning the choice of the copulas, the proposed pair construction adds the desired flexibility to Lévy copula models. We provide detailed estimation and simulation algorithms and apply the pair construction in the context of operational risk modeling.

Oliver Grothe, Stephan Nicklas
Department of Economic and Social Statistics, University of Cologne,
e-mail: nicklas@wiso.uni-koeln.de

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Nonparametric Estimation of Multivariate Extreme Value Copulas

Gordon Gudendorf* and Johan Segers

Extreme-value copulas arise in the asymptotic theory for componentwise maxima of independent random samples. An extreme-value copula is determined by its Pickands dependence function, which is a function on the unit simplex subject; see for example [1] for a survey.

Multivariate extensions of certain rank-based nonparametric estimators of the Pickands dependence function are provided, which amounts to generalizing the results in [2] to the higher-dimensional case.

The aforementioned Pickands dependence function is subject to certain shape constraints that arise from an integral transform of an underlying measure called spectral measure. These constraints are in general not satisfied by the nonparametric estimators referred to above and are enforced by replacing the initial estimator by its best least-squares approximation in the set of Pickands dependence functions having a discrete spectral measure supported on a sufficiently fine grid.

Weak convergence of the standardized estimators is demonstrated and the finite-sample performance of the estimators is investigated by means of a simulation experiment.

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Gordon Gudendorf
Université catholique de Louvain, Institut de Statistique
e-mail: gordon.gudendorf@uclouvain.be

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MEM with hierarchical Archimedean copula

Nikolaus Hautsch, Ostap Okhrin and Alexander Ristig*

Econometrics of high-frequency data aims often to model processes, which are defined on the positive support and exhibit strong persistence, e.g. conditional durations of intraday transactions. However, the dynamics of these positive valued processes are mostly described with (univariate) multiplicative error models (MEM), even though market microstructure theory suggests contemporaneous relations between several variables, e.g. trading volume and volatility. To get a better understanding of stock markets mechanisms, this paper aims to develop a multivariate MEM based on hierarchical Archimedean copula (HAC), which is estimated by a two step procedure: MEMs are calibrated to the univariate processes to obtain the standardized residuals, whose dependencies are assessed by fitting a HAC. The empirical analysis supports the hypothesis of positive and nearly stable dependence over time.

Nikolaus Hautsch,
CASE - Center for Applied Statistics and Economics, Institute for Statistics and Econometrics of Humboldt-Universität zu Berlin, Germany.
e-mail: nikolaus.hautsch@wiwi.hu-berlin.de
Ostap Okhrin,
CASE - Center for Applied Statistics and Economics, Institute for Statistics and Econometrics of Humboldt-Universität zu Berlin, Germany.
e-mail: ostap.okhrin@wiwi.hu-berlin.de
Alexander Ristig,
CASE - Center for Applied Statistics and Economics, Institute for Statistics and Econometrics of Humboldt-Universität zu Berlin, Germany.
email: ristigal@hu-berlin.de

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Ultramodular aggregation functions and copulas

Erich Peter Klement* and Radko Mesiar

An n-ary aggregation function $A : [0,1]^n \to [0,1]$ is called *supermodular* if for all $\mathbf{x},\mathbf{y} \in [0,1]^n$

$$A(\mathbf{x} \vee \mathbf{y}) + A(\mathbf{x} \wedge \mathbf{y}) \geq A(\mathbf{x}) + A(\mathbf{y}),$$

and *ultramodular* if for all $\mathbf{x},\mathbf{y},\mathbf{z} \in [0,1]^n$ with $\mathbf{x}+\mathbf{y}+\mathbf{z} \in [0,1]^n$

$$A(\mathbf{x}+\mathbf{y}+\mathbf{z}) + A(\mathbf{x}) \geq A(\mathbf{x}+\mathbf{y}) + A(\mathbf{x}+\mathbf{z}).$$

Obviously, ultramodularity implies supermodularity, and for $n = 1$ ultramodularity is equivalent to convexity.

From [3] it is known that an n-ary aggregation function $A : [0,1]^n \to [0,1]$ is ultramodular if and only if A is supermodular and if each of its one-dimensional sections is convex.

Consequently, a binary copula $C : [0,1]^2 \to [0,1]$ is ultramodular if and only if all horizontal and vertical sections are convex, i.e., if C describes the dependence structure of stochastically decreasing random vectors [4, Corollary 5.2.11].

We discuss ultramodular copulas and some methods to construct such copulas. Moreover, based on an n-ary ultarmodular aggregation function, we propose a new construction method for binary copulas [1, 2].

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Erich Peter Klement, Department of Knowledge-Based Mathematical Systems, Johannes Kepler University, Linz (Austria), e-mail: ep.klement@jku.at
Radko Mesiar, Department of Mathematics and Descriptive Geometry, Faculty of Civil Engineering, Slovak University of Technology, Bratislava (Slovakia), e-mail: mesiar@math.sk

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Singular Mixture Copulas

Dominic Lauterbach*, Dietmar Pfeifer

In [1] several geometric methods of constructing copulas are presented. One approach constructs singular copulas whose support lies in a given set. Another approach mixes an infinite family of copulas with respect to a mixing distribution.

We present a new family of copulas - the *Singular Mixture Copulas*. These copulas result from a combination of the above-mentioned methods. We begin with constructing singular copulas whose supports lie on the graphs of two given quantile functions. These copulas are then mixed with respect to a generalized beta distribution, i.e., a linear transformation of a beta distribution, resulting in a nonsingular parametric copula.

The Singular Mixture Copulas we construct have a Lebesgue density and in special cases even a closed form representation. Moreover, they have positive lower and upper tail dependence. We also provide a simple simulation algorithm. As an application we fit the copulas to flood level data. As the results show Singular Mixture Copulas provide an alternative to elliptical copulas, e.g., Gaussian and t-copulas, in modeling strongly dependent random variables.

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Dominic Lauterbach
University of Oldenburg
e-mail:dominic.lauterbach@uni-oldenburg.de

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Statistical risk evaluation of a portfolio including forward-looking stress events

Yukio Muromachi

After the recent worldwide financial crisis, it is pointed out that the stress tests, especially the test based on the forward-looking stress scenarios, are very important for the financial risk management. However, since the existing framework of the stress tests does not give any statistical meanings to their results, we cannot get fruitful informations. In this article, we propose a risk evaluation model for a portfolio including stress events with probabilities implied from market data. Our model is based on the implied copula proposed by Hull and White (2006) for pricing CDOs (Collateralized Debt Obligations), especially the synthetic CDOs. We apply the implied copula approach to the risk evaluation of a portfolio by using a framework for constructing a risk evaluation model proposed by Kijima and Muromachi (2000). In our model the "sub-filtration approach" in modeling credit risk is used, and the conditional independence of defaults is assumed. The default probabilities are modeled as stochastic variables, and adding some assumptions for simplicity, their distributions can be calibrated from the market prices of financial instruments including derivatives, for example, CDO tranche prices. Hull and White (2006) showed that even in the prosperous period there existed small probability masses in the extremely high default probability regions, which implied the latent fear of the market participants against catastrophic default events. We call such events "market-implied stress scenarios" and show that the loss distributions of CDO tranches and a bond portfolio can be constructed numerically including the effects of such events, and that they have strong influences on the risk measures such as VaR. Additionally, our numerical example shows when the extreme losses are incurred in the CDO tranches, most of the losses comes from the crash of the CDO prices, not from the actual default losses. We think that our method is one of the possible ways to connect the existing statistical models with the stress tests and to obtain useful informations.

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Yukio Muromachi
Graduate School of Social Sciences, Tokyo Metropolitan University
e-mail:muromachi-yukio@tmu.ac.jp

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Factor copula models for item response data

Aristidis K. Nikoloulopoulos* and Harry Joe

We propose factor models based on copulas for multivariate discrete data such as item response. Factor analysis is a commonly used technique for high-dimensional data to summarize the dependence structure with the number of dependence parameters being of order d, where d is the dimension. Factor copula models use bivariate copulas linking observed variables and unobserved factors, rather than conditional logistic distributions or latent multivariate normal variables. Factor copula models can allow for more probability in one or both joint tails if the linking copulas have tail dependence or tail asymmetry. The goodness-of-fit of our models is assessed with limited information methods (see for example [1]). Our general methodology is illustrated with item response data from psychometrics.

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Aristidis K. Nikoloulopoulos
School of Computing Sciences, University of East Anglia, Norwich NR4 7TJ, UK
e-mail:a.nikoloulopoulos@uea.ac.uk
Harry Joe
Department of Statistics, University of British Columbia, Vancouver, BC, Canada V6T 1Z2
e-mail:harry@stat.ubc.ca

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Bayesian estimation and model selection for high dimensional copula models

Anastasios Panagiotelis* and Michael Smith

Recent years, have seen notable growth in the development of high dimensional copulas. These can be used to flexibly model features that are commonly observed in multivariate data in finance and econometrics, such as non-linear dependence and tail dependence. Alongside developments in copula modelling, a rich literature on Bayesian estimation of these models has also emerged. In a recent paper, Smith and Khaled (2012) develop a general efficient Markov chain Monte Carlo (MCMC) scheme that can be used to estimate the parameters of almost any copula model, including in cases where some or all of the margins are discrete.

This work extends this algorithm to incorporate principles of Bayesian model selection. The margins, which are combined with a copula, depend on a potentially large number of covariates. Binary indicator variables, which parameterise whether each covariate should be included, are sampled in a computationally efficient manner. The overall modelling framework can be interpreted as a more robust and flexible version of the seemingly unrelated regression model that is prevalent in econometric modelling.

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Dr. Anastasios Panagiotelis
Monash University
e-mail:anastasios.panagiotelis@monash.edu
Prof. Michael Smith
Melbourne Business School
e-mail:mike.smith@mbs.edu

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On Spatial Contagion and mGARCH models

Marcin Pitera

Nowadays we observe the contagion not only between financial markets but also between state economies. It became one of the greatest challenges with which the global financial institutions have to deal.

In this paper we investigate the spatial (copula) contagion between two financial markets X and Y. We define contagion in similar fashion to Durante and Jaworski in [1] using concordance ordering. We study the structure of tail conditional copulas (of C_{XY}) with comparison to central set conditional copulas. We compare the values of conditional Spearman's ρ for variety of different theoretical copulas with real-stock financial data.

In particular we are interested in normal copula as it is considered as benchmark model that is contagion free. We prove useful theorems of monotonic properties of measures of concordance for Gaussian copula (with respect to conditioning parameters). We also prove that limiting copulas are of very specific type. For general contagion test, using previously proven results we propose the explicit form of the sets on which test should be performed.

We assume that time series describing financial returns follow mGARCH dynamics and investigate the contagion effect induced by such model. We consider classical multivariate GARCH models (BEKK, DCC, EDCC), static copula-GARCH, dynamic copula-GARCH and Markov Switching Copula GARCH models. When using general likelihood estimation, for real-stock market data (we select FTSE and DAX indices) we show that BEKK model provides the best fit as regards contagion modelling. It performs better than the models based on the two-step estimation procedure. We also show and explain the nature of contagion effect using above stock indices.
The presenatation is based on our work written jointly with Prof. Piotr Jaworski titled "On Spatial Contagion and mGARCH models" (preprint).

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Marcin Pitera,
Faculty of Mathematics and Computer Science, Jagiellonian University

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Modelling unbalanced clustered survival data through Archimedean copula models

Leen Prenen*, Roel Braekers, Luc Duchateau

In multivariate survival data sets, we often note that the times until an event are grouped in clusters. This induces a dependency structure on the lifetimes within a cluster. Several families of models have been developed to analyze this type of data. On the one hand, there is the frailty family in which a frailty term is introduced for all members in a cluster. We note that these shared frailty models can be applied to both balanced as well as unbalanced cluster sizes. Another family of models that takes the clustering of the observations into account are copula models. Hereby copula functions are used to model the association structure between the members of a cluster and to link the joint survival function to the different marginal survival functions. However, the practical use of copula models has been limited to clusters of equal size with a small number of members.

Using the class of Archimedean copulas, we show that it is also possible to model the clustered multivariate survival data when the clusters contain a large number of individuals or when the cluster sizes are unequal. Hereto we exploit the relationship between completely monotonic Archimedean copula generators and their Laplace transforms to rewrite the classical copula likelihood expression. The proposed model has an elegant integral structure, which naturally allows for large and unbalanced clusters. As results, we prove consistency and asymptotic normality for the copula parameters in a one-stage and a two-stage estimation procedure. In simulations studies we illustrate this consistency and asymptotic normality for the Clayton and Gumbel-Hougaard copula. Furthermore we apply these models on real life data examples in which we study the time until infection in cow udders and the time until insemination in cow herds.

Leen Prenen, I-BioStat, Hasselt University, B-3590 Diepenbeek, Belgium
e-mail:leen.prenen@uhasselt.be
Roel Braekers, I-BioStat, Hasselt University, B-3590 Diepenbeek, Belgium
Luc Duchateau, Dept. of Physiology and Biometrics, Ghent University, B-9820 Merelbeke, Belgium

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Copula-based dependence structure tests

Dominik Sznajder*, Irène Gijbels

This talk is about tests for dependence structures by means of copula functions. We discuss several general classes of dependence structures, namely quadrant dependence, tail monotonicity and stochastic monotonicity. They are of particular interest in empirical studies in finance, insurance and econometrics.

The discussed testing problems are important since a prior knowledge of a specific dependence structure is a qualitative restriction that should be taken into account in further statistical analysis, e.g., when choosing an appropriate copula function to model the dependence structure.

The considered dependence structures can be seen as the features of the underlying copula of the data generating random vector. Thus, the proposed test statistics are built as functionals of the empirical copula estimator. The statistical inference is based on resampling from a smooth constrained non-parametric copula estimator. Such an estimator is obtained by local polynomial smoothing of the initial constrained estimator and by transforming its partial derivatives by rearrangement techniques.

The proposed methodology is generic and flexible and can be applied to other dependence concepts, which can be expressed as shape constraints on the copula function.

We apply the proposed testing procedures to several real data examples available in literature.

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Dominik Sznajder
KULeuven (Belgium)
e-mail:dominik.sznajder@wis.kuleuven.be
Irène Gijbels
KULeuven (Belgium)

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Detection of arbitrage in a market with multi-asset derivatives and known risk-neutral marginals

Bertrand Tavin

In this paper we study the existence of arbitrage opportunities in a multi-asset market when risk-neutral marginal distributions of asset prices are known. We first propose an intuitive characterization of the absence of arbitrage opportunities in terms of copula functions. We then address the problem of detecting the presence of arbitrage by formalizing its resolution in two distinct ways that are both suitable for the use of optimization algorithms. The first method is valid in the general multivariate case and is based on Bernstein copulas that are dense in the set of all copula functions. The second one is easier to work with but is only valid in the bivariate case. It relies on recent results about improved Fréchet-Hoeffding bounds in presence of additional information. For both methods, details of implementation steps and numerical illustrations are provided.

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Bertrand Tavin
Université Paris 1 Panthéon Sorbonne (Laboratoire PRISM)
e-mail:btavin@gmail.com

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A circular distribution family and testing independence with uncorrelated samples

Marcus Vollmer

A one-parameter family is obtained from the bivariate circular uniform distribution (see [1], 3.1.2) using a normal distributed radius. Random variables, generated by the copula, can be used to test for conditional independence. Thus the circular distribution is uncorrelated, many independence tests will fail. This talk will introduce to the mentioned family and will present the performance of several independence tests, assessed by a simulation study. An efficient independence test, especially for small uncorrelated samples, is given by the author and introduced shortly.

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Marcus Vollmer
Department of Mathematics and Computer Science
Ernst-Moritz-Arndt-University Greifswald
e-mail:marcus.vollmer@uni-greifswald.de

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Copulas applied in the inverter case study

Marcin Zagórski*, Michał Orkisz

Development of diagnostic methods that can reduce the failure risk, identify the faulty mode or help in machine fine-tuning is in most cases a very difficult task. The dependence structure between various signals produced by a mechanical system can have in principle a very complex form. In order to reveal statistical information hidden in data, we apply the copula theory. The proposed framework is demonstrated through a case study of the inverter operational data. Particularly, for the initial analysis of dependencies we use graphical and numerical rank-based methods [1]. Moreover, for the selected pair of signals we estimate parameters of three Archimedean copulas, and validate the results against the goodness-of-fit test procedure.

In particular, the proposed method for dependence structure analysis is applied to roughly 120 samples with records of electromechanical signals, where for each sample the copula parameters are estimated. Furthermore these estimates are checked for correlation with the strength of unwanted oscillations in speed signal, which vibrations may lead to premature wear and fatigue of gears, bearings, shaft, *etc*. It is concluded that only one model reflects the dependence structure accordingly and the obtained estimates show correlation with faulty behaviour of the system.

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Marcin Zagórski
Jagiellonian University, Institute of Physics, Reymonta 4, 30-059 Kraków, Poland
e-mail:marcin.zagorski@uj.edu.pl
Michał Orkisz
ABB Corporate Research Center, Starowislna 13A, 31-038 Kraków, Poland
e-mail:michal.orkisz@pl.abb.com