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1
A copula-based approach to account for dependence in stress-strength models

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The focus of stress-strength models is on the evaluation of the probability $R = P(Y < X)$ that the stress Y experienced by a component does not exceed the strength X required to overcome it. In reliability studies, X and Y are typically modeled as independent (see [1] for an overview). Nevertheless, in many applications such an assumption may be unrealistic. This is an interesting methodological issue, especially as the estimation of R for dependent stress and strength has received only limited attention to date (e.g. [2]). This work aims to fill this gap by evaluating R taking into account the association between X and Y via a copula-based approach. We calculate a closed-form expression for R by modeling the dependence through a Farlie-Gumbel-Morgenstern copula and one of its extensions, numerical solutions for R are, instead, provided when members of Frank's copula family are employed. The marginal distributions are assumed to belong to the Burr system (i.e. Burr III, Dagum or Singh-Maddala type). In all the cases, we prove that neglect of the existing dependence leads to higher or lower values of R than is the case. As an example, using data from the 2008 wave of the Bank of Italy's Survey on Household Income and Wealth, in [3] we point out that taking into account the existing dependence improves the estimation of household financial fragility.

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2
Extended independent model based on modified product rule from the copula viewpoint

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Copula is a sophisticated way to express dependencies between variables and has been discussed especially from the theorestical perspective [1]. On the other hand, the authors have proposed a slightly different type of expression for weak dependence from the viewpoint of modified product rule motivated by the Bregman divergence which is defined based on a convex function [2]. Let $p(x)$ and $p(y)$ be the probabilities of $x \in \mathscr{X}$ and $y \in \mathscr{Y}$, and $p(x,y)$ be the joint probability of the event $(x \in \mathscr{X}, y \in \mathscr{Y})$ where $\mathscr{X}$ and $\mathscr{Y}$ be the domains of the categorical (e.g. nominal) variables X and Y. Our proposed model is given as

$$p(x,y) = u(\xi(p(x)) + \xi(p(y)) - \alpha),$$

where $u(\cdot)$ is a monotonically increasing function, $\xi(\cdot)$ is a (quasi) inverse function of $u(\cdot)$, and α is a normalizing value to hold $\sum_{x,y} u(\xi(p(x)) + \xi(p(y)) - \alpha) = 1$. Note that $p(x)$, $p(y)$ and $p(x,y)$ are probability values and are NOT "cumulative" probability values, so that this model is not equivalent to the well-known Archimedean copulas [1]. We attempt to discuss some properties of our expression of weak dependence from the viewpoint of copula.

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3
Portfolio Value-at-Risk and Expected-Shortfall in a Copula Setup

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In classical multivariate GARCH models conditional return distributions are assumed to be jointly normally distributed. Recent literature alleviates not only the constraint on the margins, but more importantly allows for a more flexible modelling of the joint dependence structure in terms of a copula function. Of special importance in such models is the calculation of portfolio risk measures which are mostly obtained via Monte Carlo simulations.
Our research tackles the analysis of portfolio risk measures without the need for simulation. By extending the fix point approach of [1] for calculating portfolio Value-at-Risk (VaR) we derive a closed form expression for VaR that is better suited for numerical treatment and higher dimensions. In this new formulation the influence of the margins is clearly separated from the influence of the dependence structure, a key feature of the copula approach. Using the same approach an alternative formulation for Expected-Shortfall (ES) is recovered in the same setting. Making use of the results in [2] we can simultaneously and efficiently compute VaR and ES in terms of a minimization problem. We contrast our approach to variants in the literature as well as Monte Carlo simulations in terms of precision and computational effort. Finally, a data example shows the applicability of the approach to real world problems.

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Construction of Multivariate Copulas in n-Boxes - Part II

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In this paper we give an alternative proof of the construction of n-dimensional ordinal sums given in Mesiar and Sempi [15], we also provide a new methodology to construct n-copulas extending the patchwork methodology of Durante, Saminger-Platz and Sarkoci in [6] and [7]. Finally, we use the gluing method of Siburg and Stoimenov [18] to give an alternative method of patchwork construction of n-copulas, which can be also used in composition with our patchwork method.

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Influence of non-classical data dependences on the minimum-based coherent risk measure.

Alexandra Karnaukhova

In modern financial mathematics it is very important to estimate risk caused by changes of the value of a portfolio during some fixed period of time. Risk measures are introduced to do it. A significant special case of such measures is a coherent measure of risk $WV@R$ (Weighted V@R) [1][2]. The object of this study is a special case of $WV@R$, called $MINV@R$ [3].

Let X be a random variable that describes a share yield. Then

$$MINV@R_N X = -Emin(X_1, ..., X_N),$$

where X_i are independent copies of X.

As it follows from the definition, there exists a simple estimate of $MINV@R$: devide observations into groups with N elements, find the minimum value of each group and then calculate its arithmetic mean.

If yields are independent, this estimate converges to $MINV@R$. But real data are not independent, so this estimate converges to another value, which we will call $MINV@R^*$.

This phenomenon has been investigated in case $N = 2$, X has the Laplace distribution. We considered different variants of non-classical dependence, for example, specified with an explicit parametric copula with cubic sections [4]. Moreover, we explored this issue with the real data of stock prices of one well-known company and of intraday gold prices.

It was established that there is a noticeable difference between $MINV@R$ and $MINV@R^*$ in both cases of correlated and non-correlated (but dependent) random variables. The research of the real data also shows the possibility of the influence of the yield's dependence on $MINV@R^*$, although the effect isn not pronounced.

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Two dimensional Ito processes coupled via conditional sampling

Marcin Krzywda

For given family of copulae $\{\mathscr{C}_t : t \geq 0\}$ we construct a stochastic process $Y = (Y^1, Y^2)$ as a function of 2-dimensional Brownian Motion B such that the join distribution of (Y_t^1, Y_t^2) is described by a copula $\mathscr{C}_t$:

$$F_t(x,y) = \mathscr{C}_t\left(\Phi(\frac{x}{\sqrt{t}}), \Phi(\frac{y}{\sqrt{t}})\right) \tag{6.1}$$

where F_t is a cumulative distribution function of Y_t and Φ denotes dcf of standard normal distribution.

To this end we exploit the conditional sampling method (cf. [1][Section 6.3]). Under some continuity and differentiation assumptions for the inverse of the partial derivative of the copula the obtained process Y is an Ito process.

Further we present a sufficient and necessary condition for Y to be a martingale, and a criterion for being a Brownian Motion.

Checking the examples for the above construction, assuming the copula is constant in time, we obtain a martingale for an independence copula and Gaussian copula, but not for Clayton or Frank copulae. The open question is whether there exists any other copula that would fulfil the assumptions and give a martingale as a result of our construction.

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Modeling time series with heavy tails and strong dependence by Gaussian copulae sequences.

Anna Mazur*, Vladimir Piterbarg

We establish the class of copula functions for which the sequence $X_k = f(\xi_k)$, where ξ_k is a standart normal random variable, belong to Maximum Domain of Attraction of Frechet Distribution with α, (MDA(α)). After that we prove limit theorem for $X_k = f(\xi_k)$ using the correlation function $\rho(k)$ of the sequence X_k when it exists (the case of $\alpha > 2$). If $\alpha < 2$, limit theorem is proven with the help of the Leadbetter's mixing conditions.

Proposition 7.1. *Suppose function $g(x)$ is continuously differentiable for all x large enough, $g(x) \to 0$ and $\frac{g'(x)}{x} \to 0$ for $x \to \infty$. Then sequence of random variables $f(\xi_k)$ belongs to MDA(α), where*

$$f(x) = C \exp(\frac{x^2}{2\alpha} + \int_0^x y g(y) dy), C > 0.$$

Denote by GC_α the space of functions f of the form above.

Proposition 7.2. *Let function $f(x) \in GC_\alpha, \alpha > 2$ and ξ_k be a Gaussian stationary sequence with zero expected value, standart variance and correlation function $r(k)$. Denote by $\rho(k)$ the correlation function of process X_k where its marginal one-dimension distribution function $F = \Phi(f^{-1}) \in MDA(\alpha)$. If either $r(k)\ln k \to 0$ or $\rho(k)\ln k \to 0$ for $k \to \infty$, then*

$$\lim_{n\to\infty} P(\max_{k=1,\ldots,n} X_k < d_n x) = e^{-e^{-x}},$$

where we can put either $d_n = F^{-1}(1 - \frac{1}{n})$ or $d_n = f(a_n)$, and $a_n = \sqrt{2\ln n} - \frac{\ln\ln n}{2\sqrt{2\ln n}}$.

If $\alpha < 2$, moments of copula may not exist, but limit theorem is still true. One may prove it using following proposition.

Proposition 7.3. *If $r(n)\ln n \to 0$ and $f \in GC_\alpha$, then Gaussian copula sequence $X_k = f(\xi_k)$ satisfies Leadbetter's mixing conditions $D(u_n)$ and $D'(u_n)$ for $u_n = x f(a_n)$ for all $x > 0$.*

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Tests and power comparisons in time-dynamic copula models

Magda Mroz

Models for multivariate time-series have become crucial for most financial applications in recent years. It is well accepted that the hypothesis of multivariate normality can be dropped in favour of more flexible dependence structures which can be easily accomplished by a copula approach. Moreover, it has become a stylized fact that correlations in financial models are not constant over time. The major objective is to test whether dependendies modelled through copulas can also be assumed time-varying.

Based on the **S**emiparametric **CO**pula-based **M**ultivariate **DY**namic model introduced in [1] and [2] a numerically tractable estimation procedure is developped. The copula parameters are estimated locally via maximum likelihood on retrospective observations of a fixed bandwidth. Subsequently, two different procedures to test the null hypothesis of parameter constancy are applied and analysed especially with respect to their ability to detect parameter inhomogeneities of various types: a Binomial test for independent time periods to detect regime changes and an extremal test for weakly dependent time periods to uncover peaks in the dependence structure.

The asymptotic local power was calculated and in case of the extremal test gave rise to an extremal distribution result for asymptotically normal quantities. In simulation studies the theoretical findings were explored and empirical analyses revealed that time-varying dependencies are present.

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Skew-t copula and its estimation : For application to risk aggregation

Toshinao Yoshiba

We improve the d variate skew t-copula approach proposed in [2] and derive log-likelihood function to estimate the parameters by maximum likelihood method. For future application, we mention risk aggregation in both market risk category and other risk categories such as credit and operational.

Based on the d variate skew t-distribution $\mathrm{St}_d(\mu, \Omega, \alpha, \nu)$ proposed in [1] with density function $f(\cdot)$ at $\mathbf{x}$,

$$f(\mathbf{x}) = 2t_d(\mathbf{x};\nu)T_1\left\{\alpha^\top W^{-1}(\mathbf{x}-\mu)\left(\frac{\nu+d}{(\mathbf{x}-\mu)^\top\Omega^{-1}(\mathbf{x}-\mu)+\nu}\right)^{1/2};\nu+d\right\},$$

the skew t-copula is given by

$$C_{\mathrm{st}}(u_1,\ldots,u_d;\tilde{\Omega},\lambda,\nu) = \mathrm{St}_d(\mathrm{St}_1^{-1}(u_1;0,1,\lambda_1,\nu),\ldots,\mathrm{St}_1^{-1}(u_d;0,1,\lambda_d,\nu);0,\tilde{\Omega},\alpha,\nu),$$

where

$$\alpha = \frac{\Lambda(\Lambda\tilde{\Omega}\Lambda-\lambda\lambda^\top)^{-1}\lambda}{\sqrt{1+\lambda^\top(\Lambda\tilde{\Omega}\Lambda-\lambda\lambda^\top)^{-1}\lambda}}, \quad \Lambda = \mathrm{diag}(\sqrt{1+\lambda_1^2},\ldots,\sqrt{1+\lambda_d^2}).$$

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