Workshop on Copula Theory and Its Applications

Preliminary Book of Abstracts
Posters

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Contribution 1
Wild Bootstrap approximation of empirical tail copulas

Axel Bücher

In a recent paper Schmidt an Stadtmüller (2006) studied some nonparametric estimators for the (upper and lower) tail copula for a sample of some bivariate random vector. The authors established weak convergence of the estimators to a centered Gaussian field with covariance structure depending on the unknown tail copula. In the present paper we propose a wild bootstrap approach to approximate the unknown limit distribution. An application of this procedure to the problem of testing for equality between two tail copulas is given for illustration.

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Contribution 2
Nonparametric bivariate modeling of petrophysical porosity-permeability well log data

Arturo Erdely* and Martín Díaz-Viera

Assessment of formation permeability is a complicated and challenging problem that plays a key role in oil reservoir forecasts and optimized reservoir management. Generally, permeability evaluation is performed using porosity-permeability relationships obtained by integrated analysis of various petrophysical parameters from cores and well logs. In carbonate double-porosity formations with complex microstructure of pore space this problem is becoming more difficult because the permeability depends on several characteristics such as: matrix porosity, secondary porosity, and, mostly, type of secondary pore system (vugs, vugs connected by channels, fractures). In this case the permeability prediction requires the adequate processing tools for statistical analysis of petrophysical data and its correlation and calibration with experimental permeability measurements.

Dependence relationships between pairs of petrophysical variables, such as permeability and porosity, are usually nonlinear and complex, and therefore those statistical tools that rely on assumptions of linearity and/or normality are not suitable in this case. The use of copulas for modeling petrophysical dependencies is not new, see Díaz-Viera and Casar-González (2005) and Díaz-Viera *et al.* (2006), where t-copulas have been used for this purpose. But expecting a single copula family to be able to model a complex bivariate dependency seems to be still too restrictive, at least for the petrophysical variables under consideration in this work. Therefore, we explore the use of the Bernstein copula, see Sancetta and Satchell (2004), and we also look for an appropriate partition of the data into subsets for which the dependence strucure was simpler to model, and then the gluing copula technique, see Siburg and Stoimenov (2008), is applied to build the copula for the whole data set.

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Contribution 3
On the out-of-sample importance of conditional dependencies for portfolio selection

Luis García

Empirical finance literature has often claimed that correlations between international equity markets are not constant. This master's thesis analyzes the importance of modelling time-varying correlations (TVC) for portfolio selection by using a multivariate copula-GARCH approach. The dependency parameters of the copula function are rendered time varying, following certain evolution depending of their previous values and historical data. We compare the out-of-sample performance of strategies that try to capture the dynamics of the correlations between equity markets against two different benchmarks: models with constant correlation parameters and the naive 1/N rule for portfolio diversification. First, we compute the Global Minimum Variance portfolio weights under each strategy and evaluate, using a bootstrap test, whether modelling TVC reduces the out-of-sample variance of returns. Second, we study the problem of an investor that aims to maximize the Sharpe Ratio of a portfolio. In this case, we test for the equality of the out-of-sample Sharpe Ratios resulting from two competing models following the approach proposed by Memmel (2003) and a bootstrap test.

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Contribution 4

Construction method for new generator based on limit conditioning of copulas

Vladimír Jágr

In the paper we introduce a new method for construction of copulas. It applies conditioning to a copula function, and the conditioning parameter is set limitly to zero. For Archimedean class, a corresponding generator with sufficient condition is provided. There can be found Archimedean copulas invariant to limit conditioning, as well as those from which we can obtain new copulas – or none at all. The three cases are illustrated in examples.

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Contribution 5
One construction method for additive archimedean generator

Monika Juráňová

We aim to seek out new construction method for archimedean copulas. It is based on enriching aditive generator φ by two more parameters, $\theta \geq 1$ and $c \in]0,\infty[$, such that

$$\varphi_{\theta,c}(x) = (c\varphi(x)+1)^{\theta} - 1,$$

for all $x \in [0,1]$. For some generators this results into a new class of Archimedean copulas. We provide also several examples to help illustration.

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Contribution 6
Copulas Induced by the Dirichlet Distribution and *B*-splines

Vladimir K. Kaishev

A considerable number of two dimensional copulas have been proposed in the literature. However, relatively few of them have truly multivariate generalizations, parametrized richly enough, so as to capture a wider range of dependence patterns, arising for example in financial and insurance risk management applications. The present paper introduces new classes of multivariate copulas, induced by the Dirichlet distribution on the simplex. Their relation to multivariate (polynomial and generalized) *B*-splines is established. It is shown that they are parametrized through the degree of the related *B*-spline, its knots and their multiplicities, which provides for a very flexible control over the dependence structure. We call such copulas *B*-spline copulas and explore some of their properties. In particular, we address evaluation of and simulation from *B*-spline copulas, showing that the latter is easy to implement. We also give formulae for the copula density. Estimation of *B*-spline copula parameters is also briefly addressed.

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Contribution 7
Bayesian model selection for D-vine pair-copula constructions

Aleksey Min* and Claudia Czado

Copulas are nowadays a standard tool for stochastic modeling in different fields of applied science. Therefore the construction of flexible multivariate copulas as well as the right choice of a copula family, i.e. model selection for copulas, have become extremely important. Recently [1] have advocated pair-copula constructions (PCC) which have been found as a most successful way of construction of multivariate copulas in many empirical studies. Using pair-copula constructions we approach the model selection problem for copulas to identify (conditional) independence, present in data, in a fully Bayesian framework. For this problem we derive and implement a reversible jump Markov chain Monte Carlo (RJ MCMC) algorithm. Further we fix building blocks of PCC's as bivariate t-copulas. However the methodology is general and can easily be extended to all known bivariate copula families. Our approach with the RJ MCMC solves model selection and estimation problems for PCC's simultaneously. The effectiveness of the developed algorithms is shown in simulations and their usefulness is illustrated in a real data application.

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Contribution 8
Interpretation of Copula on a quantum logic

Ahmed M. Al-Adilee, Olga Nánásiová

Modelling of joint distributions for non-compatible observables can be done by special functions on a quantum logic (an orthomodular lattice) L called s-maps [3]. Studying s-maps on a quantum logic helps to describe those properties of random events, which are difficult to be described by Boolean algebra. We show the meaning of the function called QL-copula, QL-co-copula on a quantum logic and we compare them to an s-map. Indeed, we show properties of QL-copula according to classical definition of copula. Finally, we show several examples of these functions by means of an s-map.

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Contribution 9
Multivariate modelling of financial data

Mária Bohdalová and Oľga Nánásiová

This paper concerns the application of copula functions in multivariate modelling of financial data. The multivariate modelling of financial data can have influence to risk analysis of the financial portfolios. Risk analysis is strongly based on the issue of correlation, or generally speaking dependence, between the returns. The copula function is used to model the dependence structure of multivariate assets. We use algorithm introduced in [3]. We compare the results obtained by this algorithm with results obtained by some known copula functions. From the comparison we show that the dependence structure between asset returns plays a more important role in modelling risk factors.

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Contribution 10
Arbitrage-free stochastic recovery framework for copula credit models

Roman Werpachowski

Copula models are standard tools for pricing structured credit derivatives which are sensitive to default correlation. The current crisis in the structured credit market had us witnessing recovery rates of senior debt ranging from 94% for Federal Home Loan Mortgage Corporation ("Freddie Mac") to 1.25% for Landsbanki bank. It highlights the need for modelling the stochastic component of recovery after default within the copula framework. In order to be arbitrage-free, the recovery model should be consistent with single name and first-to-default basket pricing. We present a computationally efficient stochastic recovery framework for copula CDO models, in the spirit of the model by M. Krekel [1]. The recovery rate is selected randomly from a small fixed set of values, with probabilities being a function of time and market factor value. We formulate the general conditions under which the model fulfils the consistency requirements and propose a particularly attractive specification of the functional form of recovery probabilities. The input to the model is the marginal distribution of recovery rates, which can be sourced from historical data or calibrated to fixed recovery CDO/credit index tranches. Correlation between recovery rates is also investigated.

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