

Dependence between components of multivariate Markov chains: Markov consistency and Markov copulae

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Modeling of evolution of dependence between processes occurring in financial markets is important. Typically, one can identify marginal statistical properties of individual processes, and then one is confronted with the task of modeling dependence between these individual processes so that the marginal properties are obeyed. We have been advocating, for some time now, to address this modeling problem via the theory of Markov consistency and Markov copulae. In this talk we shall examine the problem of existence and construction of a multivariate Markov chain with components that are given Markov chains. In this regard we shall give sufficient and necessary conditions, in terms of relevant conditional expectations, for a component of a multivariate Markov chain to be a Markov chain in its own filtration - a property called weak Markov consistency. This characterization result is proved via analysis of the semi-martingale structure of the chain.

We shall also discuss the issue of dependence between the components of a multivariate Markov chain in the context of weak Markovian consistency. Accordingly, we shall introduce and discuss the concept of weak Markov copulae. In addition, we shall examine relationship between the concepts of weak Markov consistency and weak Markov copulae, and the concepts of strong Markov consistency and strong Markov copulae that were introduced in our earlier works for Feller processes. Weak and strong Markovian copulae provide, in sense, dynamic counterparts of the classical concept of copulae in probability.

Interestingly, the strong Markovian consistency can be fully characterized analytically in terms of the generator of a multivariate Markov chain. However, this appears to be not the case for the weak Markovian consistency; here, the analytical condition needs to be supplemented with conditions imposed on the initial distribution of the chain.

We shall give an example of finite Markov chains that satisfy strong Markovian consistency, an example of a chain that satisfies weak Markovian consistency but does not satisfy strong Markovian consistency, and finally - an example of a chain that does not satisfy the weak Markovian consistency.

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This talk will be based on the recent paper by Bielecki, Jakubowski and Niewegłowski (2011).

References

- [1] Bielecki, T.R., Jakubowski, J. and Niewegłowski, M. *Intricacies of Dependence between Components of Multivariate Markov Chains: Weak Markov Consistency and Weak Markov Copulae*, submitted (2011).